Optimization of First Generation Crone Approximated Fractional-order $\text{PI}^\lambda \text{D}^\mu$ Controller by using Charged System Search

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1 Introduction.

Proportional-integral-derivative (PID) controller is one of the oldest controllers, but still it is preferred in the industry due to its simple structure and design ease. The performance of the PID controllers is increased by using more complex systems like adaptive control schemes. Recently, the performance of the PID controller is being enhanced with the aid of fractional calculus.

Fractional calculus allows designing the controller with non-integer orders of differentiation and integral. The fractional order PID is called FOPID, or $\text{PI}^\lambda \text{D}^\mu$, where additional parameters $\lambda$ and $\mu$ are the non-integer orders of the integral and derivative parts of the controller, respectively. The problem arises at this point about the implementation of a non-integer order of Laplace transform. The solution is the approximation of fractional order controllers. There exists some approximations in literature; some of them are: Crone, Carlson, Matsuda etc. Among them, Crone approximation is the most frequently preferred approximation, which is also utilized in this study.

Recent studies show that FOPID performs better performance against PID. On the other hand, despite the relative ease of tuning of PID, tuning of FOPID still remains as a challenge due to its complexity. Over the last decade, tuning process of FOPID has been realized by heuristic methods. In this study, one of the latest algorithms of this sort, the Charged System Search, is preferred.

Charged System Search (CSS) is a newly proposed method (2010), which is based on some of the fundamental principles of physics and mechanics [1]. The main idea behind this method depends on the interaction between charged particles (CP). In the algorithm, each CP (which is nothing but a search agent) can
attract/distract each other, therefore the CPs are distributed along the search space. For attraction/distraction, Coulombs and Newtons laws are applied.

2 Fractional Calculus

Fractional calculus is a branch of mathematics dealing with integration and differentiation with non-integer orders. The operator $aD_t^\alpha$ is used for non-integer order operator, where $a$ and $t$ are the limits, and $\alpha$ is the order.

$$aD_t^\alpha f(x) = \frac{1}{\Gamma(n-\alpha)} \frac{d^n}{dt^n} \int_a^t \frac{f(\tau)}{(t-\tau)^{n-\alpha}}d\tau,$$

for $(n-1 < \alpha < n)$ and $\Gamma(.)$ is Euler's Gamma function. However, transfer function notation composed from Laplace transform of the integrals, is preferred for indication of differential equations. Therefore, Laplace transform of the differentiation operator should be defined for fractional calculus. For zero initial conditions (Generally for the controller, it is assumed that the controller is initially at rest. On the other hand, it should be noted that this does not mean that the plant is initially at rest), the Laplace transform is defined as:

$$L(aD_t^\alpha f(x)) = s^\alpha F(s).$$

The fractional order of the s-domain representation should also be obtained. In the literature, there exist a couple of approximation techniques, most of which correspond to the integer order equivalent of non-integer fractional representation. Although many approximation techniques exist, in this study, Crone approximation will be preferred due to the common trends [2]. Crone approximation produces poles and zeros. The approximated unit-gain transfer function of $N^{th}$ order is

$$s^\alpha \cong \prod_{n=1}^N \frac{1 + \frac{s}{w_{z,n}}}{1 + \frac{s}{w_{p,n}}}, \alpha > 0,$$

The approximation formula given in the equation is valid for the frequency range $[w_l, w_h]$. The number of poles and zeros are the equal and given with $N$. The approximation becomes closer to the exact one by increasing $N$. On the other hand, as $N$ increases, the computation burden also increases. The unknown parameters of the equation are as follows:

$$\varepsilon = \left(\frac{w_h}{w_l}\right)^{\alpha/n}, \eta = \left(\frac{w_h}{w_l}\right)^{(1-\alpha)/n}$$

and the recursively obtained parameters are:

$$w_{z,1} = w_1 \sqrt{n}, \ w_{z,2} = w_{p,1}\eta \ \ldots \ w_{z,n} = w_{p,n-1}\eta$$

$$w_{p,1} = w_{z,1}\varepsilon, \ w_{p,2} = w_{z,2}\varepsilon \ \ldots \ w_{p,n} = w_{z,n}\varepsilon$$
3 Fractional Order PID Controller

The conventional continuous-time transfer function of the proportional-integral-differential (PID) controller is given below:

\[ G_C (s) = K_P + \frac{K_I}{s} + K_D s \] (6)

This controller is composed from Laplace transform of the differential equation, where \( s \) is the complex domain (frequency domain) abbreviation. This equation composed from three parameters \( (K_P, K_I \) and \( K_D) \) which are causes the performance of the controller. In general, the aim of the tuning process is nothing but to find these unknowns. The effects of these parameters are explained in [3]. The following transfer function represents FOPID.

\[ G_{CF} (s) = K_P + \frac{K_I}{s^\lambda} + K_D s^\mu \] (7)

This equation has two more unknowns \((\lambda \) and \( \mu) \) as the previous one. Addition of these parameters certainly increases the flexibility of the controller, but the complexity as well.

In this study, the parameters \( \lambda \) and \( \mu \) are restricted in a region given in Fig. 1.

It can be observed from the figure that, the values:
- \( \lambda=1 \) and \( \mu=1 \) correspond to the classical PID controller,
- \( \lambda=1 \) and \( \mu=0 \) correspond to the PI controller,
- \( \lambda=0 \) and \( \mu=1 \) correspond to the PD controller, and
- \( \lambda=0 \) and \( \mu=0 \) correspond to the proportional controller.

![Figure 1: The search space region for \( \lambda \) and \( \mu \)](image-url)
The aim of this study is to present a controller within the region given in Fig. 1 with better response performance compared to that of the classical PID controller and its variants. For tuning, the Charged System Search algorithm will be applied. Performance comparison of the proposed controller will be performed, and the results will be presented in details. To our belief, the outcomes of this study will create interest not only among engineers (especially the control engineers) but also among applied mathematicians dealing with differential equations, fractional calculus and optimization.

References

