Comparisons and Applications of Symmetrical Iterative Splitting Method

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Extended Abstract

The main objective of the present paper is to develop and analyze a splitting method for non-autonomous evaluation equation of the form

\[ \frac{d}{dt}u(t) = A(t)u(t), \quad t \geq 0 \]  
\[ u(0) = u_0 \in X \]

on some Banach space X. In order to solve such non-autonomous system, it is often the case that \( A(t) = T + V(t) \), where only the potential operator \( V(t) \) is time-dependent and \( T \) is the differential operator, see [1, 9].

Operator splitting is a frequently used procedure in the numerical solution of large systems of partial differential equations. One of the operator splitting methods other than the classical Trotter, Strang splitting is iterative splitting scheme which is based on first splitting the complex problem into simpler differential equations. Then each sub-equation is combined with the iterative schemes, each of which is efficiently solved with suitable integrators [7, 6, 8]. Furthermore, these sub-equations can be considered as non-homogenous initial value problems which are also discussed in [4].

Some splitting methods have already been used to find numerical solution of the different special non-autonomous systems, particularly Hamiltonian ones [5, 2, 10]. It is important to develop such numerical schemes for Hamiltonian dynamics or Schrödinger equations that preserve some important qualitative properties and geometric structure of that solution. In this study, we focus on developing the new symmetric iterative scheme. We embed the Magnus expansion [5, 3] which is a popular geometric, an attractive and widely applied method of solving explicitly time-dependent problems, in the solutions of the time dependent split subsystem of the iterative scheme.

Our main focus will be two fold: First, we develop the iterative splitting for non-autonomous problem. Second, its convergence properties are analyzed using
the concepts of stability, consistency, and order as an abstract Cauchy problem via analytic semigroup approach.

The plan of the paper as follows: In Section 2, the basic idea behind the Magnus method is summarized. In Section 3, the algorithm of the symmetric iterative scheme is presented and its convergence properties are studied. In the last section, several numerical examples are illustrated to confirm our theoretical results and efficiency of the new scheme.

References


