Forward-backward stochastic differential games and optimal portfolio under model uncertainty

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Abstract

One of the aftereffects of the financial crisis is the increased awareness of the need for more advanced modeling in mathematical finance, and a focus of attention is on the problem of model uncertainty. This presentation is motivated by a topic of this type:

We consider a stochastic system described by a general Itô-Lévy process controlled by an agent. The performance functional is expressed as the $\mathbb{Q}$-expectation of an integrated profit rate plus a terminal payoff, where $\mathbb{Q}$ is a probability measure absolutely continuous with respect to the original probability measure $\mathbb{P}$. We may regard $\mathbb{Q}$ as a scenario measure controlled by the market or the environment. If $\mathbb{Q} = \mathbb{P}$ the problem becomes a classical stochastic control problem. If $\mathbb{Q}$ is uncertain, however, the agent might seek the strategy which maximizes the performance in the worst possible choice of $\mathbb{Q}$. This leads to a stochastic differential game between the agent and the market.

Our approach is the following: We write the performance functional as the value at time $t = 0$ of the solution of an associated backward stochastic differential equation (BSDE). Thus we arrive at a (zero sum) stochastic differential game of a system of forward-backward SDEs (FBSDEs), which we study by a maximum principle approach.

We then apply the above results to study optimal portfolio and consumption problems under model uncertainty. We establish a connection between market viability under uncertainty and martingale measures.

Finally we give explicit formulas for the optimal portfolio and the optimal scenario parameter in some special cases. We obtain a model uncertainty reduction theorem, stating that any model uncertainty risk minimizing portfolio problem (of a certain type) can be reduced to model certainty problem of the same type, but with a transformed utility function.

The presentation is based on recent joint work with Agnès Sulem, INRIA, Paris.