Extended Abstract

We consider a portfolio optimization problem in an incomplete financial market consisting in a savings account with stochastic interest rate (which may suffer a shock at some random time $\tau$) and a continuous risky asset process driven by a Brownian motion $B_t$. The random time $\tau$ is not necessarily a stopping time of the Brownian filtration. We state the problem in the so-called enlarged filtration, the filtration generated by the Brownian motion and the default indicator process $H_t := 1_{\tau \leq t}$.

Our goal is to maximize $\mathbb{E}[U(X_{T^x,\pi})]$, the expected utility of the final wealth gained by a small investor, starting with the initial amount $x > 0$, over the class of admissible strategies $\pi_t$ (in a sense we make precise), which are given by the proportions of wealth invested in each asset. We take into account the cases of logarithmic utility function ($U(x) = \ln x$) and power utility function ($U(x) = x^p$, with $0 < p < 1$).

In the case of log utility the problem is solved explicitly by a direct computation, the optimal portfolio being equal to Merton’s optimal portfolio. For a power utility function, this problem is solved in two different ways:

A. by duality, case in which we cannot apply classical existence and uniqueness results of Kramkov and Schachermayer (1999), since those are valid for interest rate equal to zero. We solve the (properly defined) dual problem, on the set of equivalent martingale measures (for which we characterize the radon-Nikodym densities using a result of Kusuoka (1999) and relating afterwards this solution to the solution of the original problem;

B. by a direct approach, similar to Hu, Imkeller, Muller (2005).

We use classical results concerning Brownian Backward Stochastic Differential Equations (BSDEs), but also some new results concerning existence of a solution for BSDEs with jumps (due to Kharroubi, Lim (2011)).
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References


