The purpose of this paper is to consider the problem of an investor, who, aware of the effect of estimating errors on expected returns, adopts a robust optimization approach. Since the investor does not have direct access to the market and delegates the investment strategy to a portfolio manager, we consider a problem of delegated portfolio management with ambiguity aversion. More specifically, we are interested in analyzing the effect of ambiguity aversion on delegated portfolio choices and managerial fees. To this end, we set a simple model where a portfolio manager (not averse to ambiguity) is hired by an investor (averse to ambiguity) who pays him a fee on the final wealth produced by the selected portfolio strategy. To study the effect of the ambiguity on the fee, we assume that the manager decides to accept the job by comparing its utility to that of a similar employment where there are no restrictions derived by aversion to ambiguity. The increase on the fee paid by the investor to the manager is therefore a measure of the cost of ambiguity, or a premium for the ambiguity aversion.

We will consider two possible ways to implement an ambiguity averse strategy. In the first case, the investor asks the manager to select the portfolio by implementing a robust optimization strategy and determines a compensation for such a behavior. In the second case, the investor compensates the manager to optimize her ambiguity-averse utility, but cannot directly influence the manager’s behavior.

The problems investigated in the present paper belong to the large literature on Delegated Portfolio Management where the optimal form of a contract between the investor and manager is investigated; see Stracca (2006) [11] for a review. The seminal paper in Delegated Portfolio Management is Bhattacharya and Pfleiderer (1985) [3]. More recent references and a brief review is given in Fabretti and Herzel (2011) [6]. In the present paper, we do not investigate the optimal form
of the contract under the assumed setting. We assume an affine contract (a linear sharing rule) and compute the optimal fee. On the other hand, our approach is related to another line of research in robust portfolio selection; see e.g., [5, 7] and references therein. Robust optimization (RO) grew from the need to address data uncertainties which cannot be easily quantified in terms of probability distributions (imprecise or inexact data), and the obligation to meet certain requirements no matter what the realizations of uncertain data (hard constraints) are. Instead of assuming a probability distribution and formulating a stochastic optimization problem robust optimization confines data uncertainties into an uncertainty set, and follows a worst-case approach which takes full responsibility for all occurrences of data within the uncertainty set, an approach akin to the min-max approach of robust control. The RO methodology begins by defining suitable uncertainty sets (ellipsoidal, polyhedral) which can be justified according to the problem context, and proceeds by transforming the min-max problem into a “tractable” (that can be processed by available efficient algorithms) optimization problem. Specifically, in the context of portfolio selection, it is well-known that distribution of expected returns is not known precisely, and that portfolio composition is particularly sensitive to expected return data [1, 2, 4]. Several authors in the references [5, 7, 9] addressed this problem by applying robust optimization techniques to variants of the portfolio selection problems. The common thread is to treat the uncertainty in the returns by the choice of a suitable uncertainty set, and solve the resulting problem numerically to demonstrate the practical merits of a robust portfolio by means simulation. The choice of uncertainty sets is either motivated by statistical considerations, e.g. a regression analysis of past data returning confidence intervals, or dictated by the imperatives of ending up with a tractable (meaning convex) optimization problem.

Our setting deviates from the typical premises of RO in that we assume normally distributed returns for risky assets while we allow for ambiguity in the expected return vector. Hence our approach is more suitably akin to robust stochastic optimization; see [8, 10] for references on this subject where uncertainty is on the distribution of returns rather than on moments of a given distribution. In general, past contributions on robust portfolio selection, with the exception of Garlappi et al (2007) [7], rely on numerical solution of optimization problems whereas in this paper we obtain a closed-form robust portfolio selection rule.

Our basic setting is as follows. In a one period economy with \( n \) risky assets available for investment and following a multivariate return distribution and a riskless asset, we deal with an investor who is averse to estimation error or ambiguity in the expected return estimates of the risky assets. The investor, unable to undertake the investment herself, commissions a portfolio manager to pick stocks; the investor can force the manager to be ambiguity averse or can handle by herself the ambiguity aversion. She offers a contract which is an affine function of the final (random) wealth, and faces the problem of selecting a suitable fee to be paid to the manager (by maximizing expected final wealth after paying off the manager), which should be sufficiently high to attract the manager. The manager accepts the contract provided that his utility reservation constraint is satisfied. The main objec-
The objective of our study is to formulate a model that is simple enough to get explicit results but also sufficiently structured to address important issues such as the impact of the investor’s ambiguity aversion. This impact is easily measurable from the explicit formulae we are able to obtain.

References


