Delta Hedging in Discrete Time under Stochastic Interest Rate

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Extended Abstract

One of the most discussed assumptions of financial models, especially criticized in periods of financial turmoils, is that of market completeness, that is the perfect replication of any contingent claim by a suitable dynamic trading strategy. Theoretically, this is often achieved by ruling out any market imperfection, like illiquidity, credit risks, transactions costs, taxes, etc and by assuming the possibility of continuous time trading. Of course, real markets usually fail to satisfy most, if not all of such assumptions. One of the main challenges for financial economics is therefore to address such issues, by proposing models with less stringent hypotheses or by studying what happens when they do not hold. In this talk we want to focus on the impossibility of trading continuously in time. Even if all other assumptions of the model are satisfied, the inherent discreteness of trading times is a source of market incompleteness in the real world. The aim of the paper is to efficiently evaluate the impact of trading in discrete time on the final goal of the strategy.

The object of our investigation is the ex-ante assessment of the performances of dynamic trading strategies. Probably, the most notable instance of such problem is measuring the hedging error of a strategy, based on a liquid assets, that tries to hedge a future liability. Problems of such kind arise when replicating either a claim using futures contracts, or a payoff of a derivative security with a delta hedging strategy based on the underlying asset, and in any case when a dynamic strategy is adopted. Ex-ante, a possible way to measure the performance of a strategy is by evaluating expected value and variance of its hedging error. This is usually done by approximations or by Monte Carlo simulations. The approach we propose, based on Laplace transforms, allows to efficiently perform such computations for a very general class of models.

We consider a market model driven by continuous time affine processes, in which by definition the conditional characteristic function is an exponential of an affine function of the state variables (see Duffie et al. (2000) for a formal definition and properties of affine models). In this framework, Angelini and Herzel (2009,
Angelini and Herzel (2012) provide semi-closed formulas for the efficient computation of expected value and variance of the hedging error for a quite general class of strategies, called "affine", that includes the popular Delta hedging strategy. Such formulas are obtained using a Laplace transform approach, that is based on the idea of writing the payoff of the contingent claim as an inverse Laplace transform. An important feature of the result is that one can study different type of mispecification. For instance, it is possible to analyse the performance of the standard Black-Scholes Delta strategy when the underlying asset is driven by a process which is not log-normal, like in a stochastic volatility model.

Angelini and Herzel (2012) made the simplifying assumption of deterministic interest rates. In the present work, we extend the analysis to the case of stochastic interest rate. Such extension gives us the opportunity to study the hedging problem in a more general and realistic model. For example, we can study the influence of various interest rate parameters on the hedging performances and also the effect of assuming that the interest rate is deterministic when in fact it is stochastic. For the sake of clearness, we consider a simple two-dimensional affine model, where the underlying evolves according to the Black-Scholes dynamics, while the short-term interest rate follows the process of the Vasicek model, and the stock and the interest rate may be correlated. This is a particular case of a model considered in van Haastrecht et al. (2009) to price long-term derivatives. Notice that if the Cox, Ingersoll and Ross model were used for the interest rate, the resulting two-dimensional model would be affine only in case of zero correlation. Within this model, we implement two types of Delta strategies: the correct model strategy that takes into account the stochasticity of the interest rate, which may be called the model Delta, and the plain Black-Scholes Delta with deterministic rate. We show that the differences between the two strategies may be relevant, mainly depending on the correlation and on the relation between the volatility of the risky asset and that of the interest rate. We conclude that the standard Black-Scholes strategy, still very used by practitioners, may be unappropriate because it may lead to a variance of the error much higher, in relative terms, to that obtained with the correct Delta, especially when the volatility of the interest rates is comparable with that of the stock.

As a final application, we study the Delta hedging for an interest rate option in the Cox, Ingersoll and Ross (1985) model, showing numerical illustrations in the case that the objective measure under which the short rate evolves differs from the risk-neutral measure used to implement the strategy.

References


