Stress Intensity factors of a Circumferential Crack in an Isotropic Curved Beam Using Modified Mapping-Collocation Method

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Extended Abstract

Curved composite beams are frequently encountered in aero-structures and wind turbine blades. To investigate delamination in these structural components, two curved beams attached by a weak interface can be used. Failure of composite L-shaped brackets which connect wing skin to its corresponding ribs, is a subject of current interest [1].

For purpose of Linear Elastic Fracture Mechanics (LEFM), it is necessary to determine the value of the Stress Intensity Factor (SIF) for determining the singularity at the crack tip. An isotropic homogeneous beam can serve as a model to determine the stress intensity factor in these geometries.

There are various approaches to determine the Stress Intensity Factors. The Modified Mapping-Collocation (MMC) method [2] is a semi-analytic method which combines Complex Analysis with the Boundary Collocation method.

Complex Analysis allows the powerful tools of Analytic Function Theory and Conformal Mapping to be applied to plane elasticity Boundary Value Problem (BVP). Utilizing the Cauchy-Reimann Conditions the boundary condition equations are formulated in terms of analytic functions \(\phi(z)\) and \(\psi(z)\). By a proper conformal mapping function, (using the concept of Analytic Extension (Continuation) Arguments) [3], it becomes possible to re-formulate in terms of \(\psi(z)\) only. The mapping function is found to be:

\[ z = h(\zeta) = R_1 e^{i[\beta(\zeta+\zeta^{-1}) - ik - \pi]} \]

where \(R_1\) is inner radius; \(\beta\) and \(k\) are geometrical parameters too.

The Boundary Collocation method is a numerical method which tends to satisfy the boundary condition equations by collocating (or assigning) error on the
boundaries (to be minimized). Considering symmetry with respect to the imaginary axis; *Laurent Series Expansion* for $\phi(\zeta)$ takes the form:

$$
\phi(\zeta) = \sum_{n=-M}^{N} (iA_{2n+1}\zeta^{2n+1} + A_{2n}\zeta^{2n})
$$

where $A_{2n+1}$ and $A_{2n}$ are purely real. These coefficients are the unknowns of the problem. The expansion above should be substituted in the *Boundary Condition Equations* in $\zeta$-Plane to construct a linear system of equations. The MMC method offers *local force resultant* and *local moment* on the boundaries (which leads to redundancy) to be considered in addition to the conventional stress boundary condition equations. Thus the resulting system is to be solved in a *least-square* sense. Once the system solved; the stress field and consequently stress intensity factors can be yielded.

The effects of beam thickness and crack arc size on SIF (for both Opening and Sliding Modes) are investigated. The results are validated using *Finite Element Method (FEM)* software.

**References**

