The Alexia solution for Talmudic problems

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Extended Abstract

1 Introduction.

This study is based on the results of Guiasu (2011) who solves the three ancient problems, namely the bankruptcy problem, the contested garment problem and the rights arbitration problem, from the 2000 year old Babylonian Talmud by using the Shapley value (Shapley (1953)) from cooperative game theory. The main objective of Guiasu (2011) is to show that the Shapley value can be used for justifying the ancient solutions given to all these three problems viewed as cooperative n-person games.

Tijs et al. (2011) introduce the average lexicographic (Alexia) value, a value which averages the lexicographic maxima of the core (Gillies (1959)), for games with a non-empty core and show that the Alexia value coincides with the Shapley value for the class of convex games. Further, Curiel, Maschler and Tijs (1987) show that bankruptcy games are convex.

In the sequel we show that the Alexia value can also be an alternative to the Shapley value for the solution of these problems, and note that in general the most popular one-point solution concepts such as the Shapley value for general TU-games do not provide a core-element as a solution, but the Alexia value provides a core-element as a solution for all games with a non-empty core.

2 The model and the solution

We first consider the bankruptcy problem which is known as Talmudic problem of three wives. In the story a man married with three women and promised them in their marriage contract the sum of $d_1 = 100, d_2 = 200$ and $d_3 = 300$ units of money after his death. But, the estate $E$ was less then 600 units. The division of the estate among the three wives is as follows: for is $E = 100$, the wives get $33\frac{1}{3}$,
33\frac{1}{3}, 33\frac{1}{3}; for E = 200 the wives get 50, 75, 75 and for E = 300 the wives get 50, 100, 150, respectively (cf. Gura (2009)).

The second story is called the contested garment problem. The two hold a garment and one of them wants all, and the other half of it. Then the division of the garment is three quarters for the former one and one quarter for the latter one (cf. O’Neill (1982)).

Finally, we look at the right arbitration problem. The story is based on a father and his four sons. The father Jacob willed different units of money from his estate to his four sons. So after his death, the four sons produced different deeds and all of them bear the same date. The son Reuben produced a deed duly witnessed that Jacob willed to him the entire estate on his death, the son Simeon produced a deed that his father willed to him half of the estate, the son Levi produced a deed giving him one third and the son Judah produced a deed giving him one forth. Assuming that the estate is \( E = 120 \) the division between the sons is \( 7\frac{1}{2}, 10\frac{5}{6}, 20\frac{5}{6} \) and \( 80\frac{5}{6} \), respectively (cf. O’Neill (1982)).

Following the steps of Guiasu (2011), we show that the division of the estate for these three ancient problems may be obtained by using the Alexia value, if the bankruptcy problem is viewed as being a cumulative game and the rights arbitration problem is viewed as being a maximal game. In a cumulative TU game the members are not willing to reach a compromise and share their claims. So the values of the characteristic function for the coalitions are calculated additively. On the other hand in a maximal TU game the members of the coalition are willing to reach a compromise and share their claims. For this reason the values of the characteristic function for the proper coalitions are calculated by taking into account the maximum claim.

## 3 Concluding remarks

In this study we try to give an alternative solution, the Alexia value from cooperative game theory, for to solve the three ancient bankruptcy situations from the 2000 year old Babylonian Talmud. Since each situation is modeled by using bankruptcy games, we used the result of Curiel Maschler and Tijs (1987) that each bankruptcy game is convex and the result of Tijs et al. (2011) that for each convex game the Shapley value and the Alexia value are equal.

The Alexia value is an interesting alternative to the Shapley value because the Alexia value provides a core-element as a solution for all games with a non-empty core. Moreover it can be seen as a run-to-the-core rule for games with a non-empty core. To be more precise for every lexinal players are running to the core according to a certain order where every player takes the maximum he can obtain within the subset of the core that remains after the players before him have made their respective choices. Further, the Alexia value combines two often applied arguments with respect to choosing an allocation: using orderings of the players, and at the same time respecting the fairness criterion of the core. Hence, the Alexia
value combines the attractive properties of the Shapley value.

References


